FORM TP 2010227

TEST CODE **02134020**

MAY/JUNE 2010

CARIBBEAN EXAMINATIONS COUNCIL

ADVANCED PROFICIENCY EXAMINATION

PURE MATHEMATICS

UNIT 1 – PAPER 02

ALGEBRA, GEOMETRY AND CALCULUS

2 ½ hours

20 MAY 2010 (p.m.)

This examination paper consists of **THREE** sections: Module 1, Module 2 and Module 3.

Each section consists of 2 questions.
The maximum mark for each Module is 50.
The maximum mark for this examination is 150.
This examination consists of 7 printed pages.

INSTRUCTIONS TO CANDIDATES

1. **DO NOT** open this examination paper until instructed to do so.
2. Answer **ALL** questions from the **THREE** sections.
3. Write your solutions, with full working, in the answer booklet provided.
4. Unless otherwise stated in the question, any numerical answer that is not exact **MUST** be written correct to three significant figures.

Examination Materials Permitted

Graph paper (provided)
Mathematical formulae and tables (provided) – **Revised 2009**
Mathematical instruments
Silent, non-programmable, electronic calculator

SECTION A (Module 1)

Answer BOTH questions.

1. (a) Find the values of the constant p such that $x - p$ is a factor of

$$f(x) = 4x^3 - (3p + 2)\,x^2 - (p^2 - 1)\,x + 3.$$ **[5 marks]**

(b) Solve, for x and y, the simultaneous equations

$$\log (x - 1) + 2 \log y = 2 \log 3$$

$$\log x + \log y = \log 6.$$ **[8 marks]**

(c) Solve, for $x \in \mathbf{R}$, the inequality

$$\frac{2x - 3}{x + 1} - 5 > 0.$$ **[5 marks]**

(d) By using $y = 2^x$, or otherwise, solve

$$4^x - 3\,(2^{x+1}) + 8 = 0.$$ **[7 marks]**

Total 25 marks

2. (a) (i) Use the fact that $S_n = \sum_{r=1}^{n} r = \frac{1}{2} n(n+1)$ to express

$$S_{2n} = \sum_{r=1}^{2n} r \text{ in terms of } n.$$ **[2 marks]**

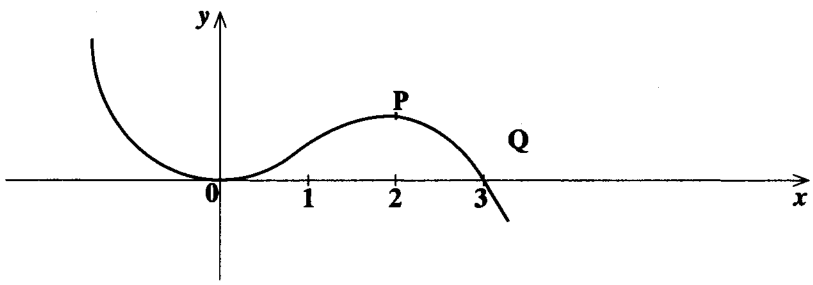
(ii) Find constants p and q such that

$$S_{2n} - S_n = pn^2 + qn.$$ **[5 marks]**

(iii) Hence, or otherwise, find n such that

$$S_{2n} - S_n = 260.$$ **[5 marks]**

(b) The diagram below **(not drawn to scale)** shows the graph of $y = x^2(3 - x)$. The coordinates of points P and Q are (2, 4) and (3, 0) respectively.



(i) Write down the solution set of the inequality $x^2(3 - x) \leq 0$. **[4 marks]**

(ii) Given that the equation $x^2(3 - x) = k$ has three real solutions for x, write down the set of possible values for k. **[3 marks]**

(iii) The functions f and g are defined as follows:

$$f : x \rightarrow x^2(3 - x), \quad 0 < x < 2$$
$$g : x \rightarrow x^2(3 - x), \quad 0 < x < 3$$

By using (b) (ii) above, or otherwise, show that

a) f has an inverse

b) g does NOT have an inverse. **[6 marks]**

Total 25 marks

SECTION B (Module 2)

Answer BOTH questions.

3. (a) The vectors **p** and **q** are given by

$$\mathbf{p} = 6\mathbf{i} + 4\mathbf{j}$$
$$\mathbf{q} = -8\mathbf{i} - 9\mathbf{j}.$$

(i) Calculate, in degrees, the angle between **p** and **q**. **[5 marks]**

(ii) a) Find a non-zero vector **v** such that $\mathbf{p}.\mathbf{v} = 0$.

b) State the relationship between **p** and **v**. **[5 marks]**

(b) The circle C_1 has $(-3, 4)$ and $(1, 2)$ as endpoints of a diameter.

(i) Show that the equation of C_1 is $x^2 + y^2 + 2x - 6y + 5 = 0$. **[6 marks]**

(ii) The circle C_2 has equation $x^2 + y^2 + x - 5y = 0$. Calculate the coordinates of the points of intersection of C_1 and C_2. **[9 marks]**

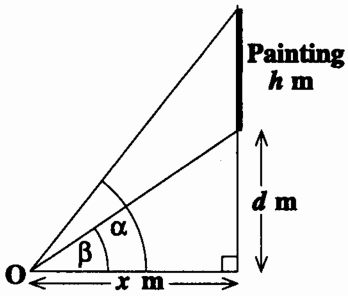
Total 25 marks

4. (a) (i) Solve the equation $\cos 3A = 0.5$ for $0 \leq A \leq \pi$. **[4 marks]**

(ii) Show that $\cos 3A = 4\cos^3 A - 3\cos A$. **[6 marks]**

(iii) The THREE roots of the equation $4p^3 - 3p - 0.5 = 0$ all lie between -1 and 1. Use the results in (a) (i) and (ii) to find these roots. **[4 marks]**

(b) The following diagram, **not drawn to scale**, represents a painting of height, h metres, that is fastened to a vertical wall at a height of d metres above, and x metres away from, the level of an observer, O.



The viewing angle of the painting is $(\alpha - \beta)$, where α and β are respectively the angles of inclination, in radians, from the level of the observer to the top and base of the painting.

(i) Show that $\tan(\alpha - \beta) = \dfrac{hx}{x^2 + d(d+h)}$. **[6 marks]**

(ii) The viewing angle of the painting, $(\alpha - \beta)$, is at a maximum when $x = \sqrt{h(d+h)}$. Calculate the maximum viewing angle, in radians, when $d = 3h$.

[5 marks]

Total 25 marks

SECTION C (Module 3)

Answer BOTH questions.

5. (a) Find

(i) $\lim_{x \to 3} \dfrac{x^2 - 9}{x^3 - 27}$ **[4 marks]**

(ii) $\lim_{x \to 0} \dfrac{\tan x - 5x}{\sin 2x - 4x}$. **[5 marks]**

(b) The function f on **R** is defined by

$$f(x) = \begin{cases} 3x - 7, & \text{if } x > 4 \\ 1 + 2x, & \text{if } x \leq 4. \end{cases}$$

(i) Find

a) $\lim_{x \to 4^+} f(x)$ **[2 marks]**

b) $\lim_{x \to 4^-} f(x)$. **[2 marks]**

(ii) Deduce that $f(x)$ is discontinuous at $x = 4$. **[2 marks]**

(c) (i) Evaluate $\int_{-1}^{1} \left[x - \frac{1}{x}\right]^2 \mathrm{d}x$. **[6 marks]**

(ii) Using the substitution $u = x^2 + 4$, or otherwise, find

$$\int x \sqrt{x^2 + 4}\, \mathrm{d}x.$$ **[4 marks]**

Total 25 marks

6. (a) Differentiate with respect to x

(i) $y = \sin(3x + 2) + \tan 5x$ **[3 marks]**

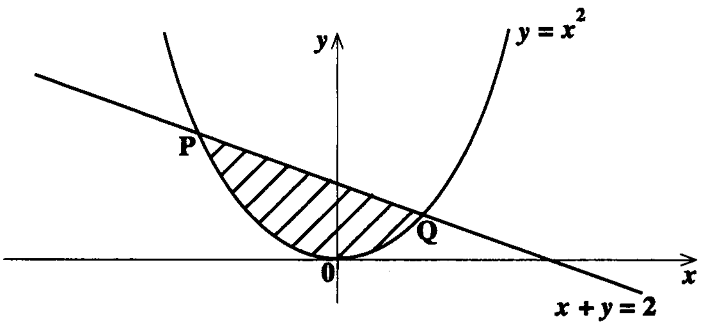
(ii) $y = \dfrac{x^2 + 1}{x^3 - 1}$. **[4 marks]**

(b) The function $f(x)$ satisfies $\int_1^4 f(x)\,dx = 7$.

(i) Find $\int_1^4 [3f(x) + 4]\,dx$. **[4 marks]**

(ii) Using the substitution $u = x + 1$, evaluate $\int_0^3 2f(x + 1)\,dx$. **[4 marks]**

(c) In the diagram below **(not drawn to scale)**, the line $x + y = 2$ intersects the curve $y = x^2$ at the points P and Q.



(i) Find the coordinates of the points P and Q. **[5 marks]**

(ii) Calculate the area of the shaded portion of the diagram bounded by the curve and the straight line. **[5 marks]**

Total 25 marks

END OF TEST