CARIBBEAN EXAMINATIONS COUNCIL

ADVANCED PROFICIENCY EXAMINATION

PURE MATHEMATICS

UNIT 1 – Paper 02

ALGEBRA, GEOMETRY AND CALCULUS

2 hours 30 minutes

10 MAY 2012 (p.m.)

This examination paper consists of **THREE** sections: Module 1, Module 2 and Module 3.

Each section consists of 2 questions.
The maximum mark for each Module is 50.
The maximum mark for this examination is 150.
This examination consists of 6 printed pages.

READ THE FOLLOWING INSTRUCTIONS CAREFULLY.

1. **DO NOT** open this examination paper until instructed to do so.
2. Answer **ALL** questions from the **THREE** sections.
3. Write your solutions, with full working, in the answer booklet provided.
4. Unless otherwise stated in the question, any numerical answer that is not exact **MUST** be written correct to three significant figures.

Examination Materials Permitted

Graph paper (provided)
Mathematical formulae and tables (provided) – **Revised 2012**
Mathematical instruments
Silent, non-programmable, electronic calculator

DO NOT TURN THIS PAGE UNTIL YOU ARE TOLD TO DO SO.

SECTION A (Module 1)

Answer BOTH questions.

1. (a) The expression $f(x) = 2x^3 - px^2 + qx - 10$ is divisible by $x - 1$ and has a remainder -6 when divided by $x + 1$.

Find

(i) the values of the constants p and q **[7 marks]**

(ii) the factors of $f(x)$. **[3 marks]**

(b) Find positive integers x and y such that

$$(\sqrt{x} + \sqrt{y})^2 = 16 + \sqrt{240}.$$ **[8 marks]**

(c) (i) Solve, for real values of x, the inequality

$$|3x - 7| \leq 5.$$ **[5 marks]**

(ii) Show that no real solution, x, exists for the inequality $|3x - 7| + 5 \leq 0$. **[2 marks]**

Total 25 marks

2. (a) The function f on **R** is defined by

$$f: x \rightarrow x^2 - 3.$$

(i) Find, in terms of x, $f(f(x))$. **[3 marks]**

(ii) Determine the values of x for which

$$f(f(x)) = f(x + 3).$$ **[6 marks]**

(b) The roots of the equation $4x^2 - 3x + 1 = 0$ are α and β.

Without solving the equation

(i) write down the values of $\alpha + \beta$ and $\alpha\beta$ **[2 marks]**

(ii) find the value of $\alpha^2 + \beta^2$ **[2 marks]**

(iii) obtain a quadratic equation whose roots are $\frac{2}{\alpha^2}$ and $\frac{2}{\beta^2}$. **[5 marks]**

(c) Without the use of calculators or tables, evaluate

(i) $\log_{10}\left(\frac{1}{3}\right) + \log_{10}\left(\frac{3}{5}\right) + \log_{10}\left(\frac{5}{7}\right) + \log_{10}\left(\frac{7}{9}\right) + \log_{10}\left(\frac{9}{10}\right)$ **[3 marks]**

(ii) $\sum_{r=1}^{99} \log_{10}\left(\frac{r}{r+1}\right)$. **[4 marks]**

Total 25 marks

SECTION B (Module 2)

Answer BOTH questions.

3. (a) (i) Given that $\cos(A+B) = \cos A \cos B - \sin A \sin B$ and $\cos 2\theta = 2\cos^2\theta - 1$, prove that

$$\cos 3\theta \equiv 2\cos\theta\left[\cos^2\theta - \sin^2\theta - \frac{1}{2}\right].$$ **[7 marks]**

(ii) Using the appropriate formula, show that

$$\frac{1}{2}[\sin 6\theta - \sin 2\theta] \equiv (2\cos^2 2\theta - 1)\sin 2\theta.$$ **[5 marks]**

(iii) Hence, or otherwise, solve $\sin 6\theta - \sin 2\theta = 0$ for $0 \leq \theta \leq \frac{\pi}{2}$. **[5 marks]**

(b) Find ALL possible values of $\cos\theta$ such that $2\cot^2\theta + \cos\theta = 0$. **[8 marks]**

Total 25 marks

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4. (a) (i) Determine the Cartesian equation of the curve, **C**, defined by the parametric equations $y = 3 \sec \theta$ and $x = 3 \tan \theta$. **[5 marks]**

(ii) Find the points of intersection of the curve $y = \sqrt{10x}$ with **C**. **[9 marks]**

(b) Let **p** and **q** be two position vectors with endpoints (–3, 4) and (–1, 6) respectively.

(i) Express **p** and **q** in the form $x\mathbf{i} + y\mathbf{j}$. **[2 marks]**

(ii) Obtain the vector $\mathbf{p} - \mathbf{q}$. **[2 marks]**

(iii) Calculate $\mathbf{p} \cdot \mathbf{q}$. **[2 marks]**

(iv) Let the angle between **p** and **q** be θ. Use the result of (iii) above to calculate θ in degrees. **[5 marks]**

Total 25 marks

SECTION C (Module 3)

Answer BOTH questions.

5. (a) (i) Find the values of x for which $\frac{x^3+8}{x^2-4}$ is discontinuous. **[2 marks]**

(ii) Hence, or otherwise, find

$$\lim_{x \to -2} \frac{x^3+8}{x^2-4}.$$ **[3 marks]**

(iii) By using the fact that $\lim_{x \to 0} \frac{\sin x}{x} = 1$, or otherwise, find,

$$\lim_{x \to 0} \frac{2x^3+4x}{\sin 2x}.$$ **[5 marks]**

(b) The function f on **R** is defined by

$$f(x) = \begin{cases} x^2+1, & x > 1, \\ 4+px, & x < 1. \end{cases}$$

(i) Find

a) $\lim_{x \to 1^+} f(x)$ **[2 marks]**

b) the value of the constant p such that $\lim_{x \to 1} f(x)$ exists. **[4 marks]**

(ii) Hence, determine the value of $f(1)$ for f to be continuous at the point $x = 1$. **[1 mark]**

(c) A chemical process in a manufacturing plant is controlled by the function

$$M = ut^2 + \frac{v}{t^2}$$

where u and v are constants.

Given that $M = -1$ when $t = 1$ and that the rate of change of M with respect to t is $\frac{35}{4}$ when $t = 2$, find the values of u and v.

[8 marks]

Total 25 marks

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6. (a) (i) Given that $y = \sqrt{4x^2 - 7}$, show that $y\dfrac{dy}{dx} = 4x$. **[3 marks]**

(ii) Hence, or otherwise, show that

$$y\frac{d^2y}{dx^2} + \left(\frac{dy}{dx}\right)^2 = 4.$$ **[3 marks]**

(b) The curve, C, passes through the point $(-1, 0)$ and its gradient at the point (x, y) is given by

$$\frac{dy}{dx} = 3x^2 - 6x.$$

(i) Find the equation of C. **[4 marks]**

(ii) Find the coordinates of the stationary points of C. **[3 marks]**

(iii) Determine the nature of EACH stationary point. **[3 marks]**

(iv) Find the coordinates of the points P and Q at which the curve C meets the x-axis. **[5 marks]**

(v) Hence, sketch the curve C, showing

a) the stationary points

b) the points P and Q. **[4 marks]**

Total 25 marks

END OF TEST

IF YOU FINISH BEFORE TIME IS CALLED, CHECK YOUR WORK ON THIS TEST.